PURE MATHEMATICS
(Three hours and a quarter)

(The first 15 minutes of the examination are for reading the paper only. Candidate must NOT start writing during this time).

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*Answer **Question 1** from Section **A** and **10** questions from section **B**. All working, including rough work, should be done on the same sheet adjacent to the rest of the answer. The intended marks for the questions or parts of questions are given in brackets* [ ].

Mathematical formulae are given at the end of this question paper. The use of calculator (fx-82 / fx-100) is only allowed.

Section A (2 × 15 = 30)
(Answer ***ALL*** questions)

Directions: *Read the following questions carefully. For each question there are four alternatives A, B, C and D. Choose the correct alternative and write it in your answer sheet.*

Question 1

i). The value of $\cos 855°$ is

A $\frac{1}{\sqrt{3}}$ **B** $\frac{2}{\sqrt{3}}$ **C** $\frac{-1}{2}$ **D** $\frac{-1}{\sqrt{2}}$

ii). If $y = \cos ec\frac{2}{3}x$ then $\frac{dy}{dx} =$

A $\sec\frac{3}{2}x$ **B** $-\frac{3}{2}\cot\frac{2}{3}x$ **C** $-\frac{2}{3}\cos ec\frac{2}{3}x\cot\frac{2}{3}x$ **D** $-\sec\frac{2}{3}x\tan\frac{2}{3}x$

iii). The remainder when $5x^3 - 8x^2 + 3x - 4$ divided by $x - 1$ is

A 14 **B** -4 **C** -12 **D** 4

iv). The approximate change in $\frac{1}{x}$, when $x = 1$ and $\delta x = 0.2$ is

A -0.2 **B** -2 **C** 0.2 **D** 2

v). The value of $\frac{\sin^2 60° + \cos^2 60°}{\sec^2 75° - \tan^2 75°}$ is

A -1 **B** -2 **C** ∞ **D** 1

vi). Sum of 20 terms of the progression 1, 3, 5, 7, …… is

A 400 **B** 250 **C** 350 **D** 600

vii). A particle moves along a straight line according to the law, $s = at^2 + 2bt + c$, then the acceleration is

A Zero **B** Constant **C** Square of Velocity **D** Double of displacement.

viii). $\lim_{x\to 4^+} \frac{x-4}{|x-4|} = ...$

A 0 **B** 1 **C** 8 **D** -4

ix). Slope of the curve $y = 2x^2 - 1$ at $x = 1$ is

A 4 **B** 6 **C** 8 **D** -4

x). If $\log_x \frac{1}{36} = \frac{-2}{3}$, then the value of *x* is

A 36 **B** 196 **C** 216 **D** 66

xi). $\int \sin^2 x \, dx = ...$

A $\cos^2 x + c$ **B** $\frac{x}{2} - \frac{\sin 2x}{4} + c$ **C** $-\frac{\sin 2x}{4} + c$ **D** $\sin 2x + c$

xii). The equation of the straight line through the point $(-1, -5)$ and slope $\frac{-6}{11}$ is

A $6x + 11y + 61 = 0$ **B** $16x + y - 16 = 0$ **C** $y + 6x - 16 = 0$ **D** $11x + 6y - 61 = 0$

xiii). If $f(x) = \frac{7}{4}x^2$, then the value of $f'\left(\frac{1}{7}\right)$ is

A $\frac{7}{2}$ **B** $\frac{1}{2}$ **C** $\frac{1}{7}$ **D** $\frac{3}{2}$

xiv). Standard deviation first five prime number is

A 2.3 **B** 3.4 **C** 3.2 **D** 4.3

xv). The value of $\sin 80° \cos 20° - \cos 80° \sin 20°$ is

A $\frac{1}{\sqrt{3}}$ **B** $\frac{\sqrt{3}}{2}$ **C** $\frac{1}{2}$ **D** $\frac{-1}{2}$

Section B (70 Marks)

*Answer any **TEN** questions. All questions in this section have equal marks. Unless otherwise stated, you may round answers to 2 decimal places.*

Question 2

a) Prove that $\frac{1+\sin\theta}{1-\sin\theta} = (\sec\theta + \tan\theta)^2$ [3]

b) If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, Prove that $2x\frac{dy}{dx} + y = 2\sqrt{x}$ [4]

Question 3

a) Prove that $\cos 3x = 4\cos^3 x - 3\cos x$ [4]

b) Find the median of the following distribution: [3]

Marks	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Students	3	5	20	10	5

Question 4

a) If $f(x)=\frac{x+2}{x-2}$, for all $x\neq 2$, find $f'(-2)$. [3]

b) Find the quartile deviation and coefficient of quartile deviation from the following frequency distribution. [4]

Wages	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of workers	60	45	120	25	90	80	120	60

Question 5

a) Find the equation to the straight line passing through the point $(4,-5)$ and (a) parallel (b) perpendicular to the line $3x+4y+5=0$. [4]

b) Evaluate: $\int \frac{3x^4+7x-11}{x^3}dx$ [3]

Question 5

a) If $(7,a),(-5,2)$, and $(3,6)$ are collinear, find a. [3]

b) Evaluate: $\int \frac{x}{(x^2-1)^{3/2}}\,dx$ [4]

Question 6

a) Simplify: $\frac{\sin(90-\theta)\sec(180-\theta)\sin(-\theta)}{\sin(180+\theta)\cot(360-\theta)\cos ec(90+\theta)}$ [4]

b) If $f(x)=\log\left(\frac{1-x}{1+x}\right)$, show that $f(a)+f(b)=f\left(\frac{a+b}{1+ab}\right)$ [3]

Question 7

a) The standard deviation of the numbers 2, 3, 11, x is $3\frac{1}{2}$.

Calculate the possible values of x. [4]

b) Prove that $\sqrt{\frac{1-\cos x}{1+\cos x}}=\tan\frac{x}{2}$ [3]

Question 8

a) If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, find the value of k. [4]

b) A circular plate expands when heated from a radius of 5 cm to 5.06 cm. Find the approximate increase in area. [3]

Question 9

a) In what ratio is the line joining points $(3,-6)$ and $(-6,8)$ divided by y- axis. [3]

b) If $\sin\theta=\frac{-1}{2}$, θ in quadrant IV, then find the values of $\sin 2\theta$ and $\cos 2\theta$. [4]

Question 10

a) Factorize: $x^3 - 7x + 6$ [4]

b) If $f'(x) = x^4 - \frac{2}{x^3} + x^2$, then find $f(x)$. [3]

Question 11

a) Evaluate : $\lim_{x \to \pi/2} \frac{1 + \cos 2x}{(\pi - 2x)^2}$ [4]

b) Differentiate w.r.to x : $x^3 \sec 5x$ [3]

Question 12

a) Differentiate $\cos x$ w.r.to x., by using first principle. [4]

b) Prove that $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1+x^2}}{x}$ [3]

Question 13

a) Find the variance for the following distribution: [4]

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	5	12	15	20	10	4	2

b) Evaluate: $\int \frac{\sin x}{\cos^2 x} dx$ [3]

Question 14

a) Examine whether $f(x) = \frac{e^{|\sin x|}}{x^3}$ is an odd or even function [3]

b) Resolve into partial fraction: $\frac{2x^2 + 7x + 23}{(x+3)^2 (x-1)}$ [4]

Mathematical formulae

Algebra

(1) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(2) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

(3) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(4) $x^{-n} = \dfrac{1}{x^n}, x \neq 0$

(5) $t_n = a + (n-1)d$

(6) $t_n = ar^{n-1}$

(7) $S_n = \dfrac{n}{2}\left(2a + (n-1)d\right)$

(8) $S_n = \dfrac{a(1-r^n)}{1-r}$, where $r < 1$

(9) $S_n = \dfrac{a(r^n - 1)}{r-1}$, where $r > 1$

Coordinate Geometry

(1). $(y - y_1) = m(x - x_1)$

(2). $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(3). $(x, y) = \left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

(4). Intercept Form: $\dfrac{x}{a} + \dfrac{y}{b} = 1$

(5). Normal Form: $x\cos\alpha + y\sin\alpha$

(6). Angle between two lines:

$\tan\theta = \left|\dfrac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2}\right|$

(7). Distance of a point from a line

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Statistics

(1). Mean, $\bar{X} = A + \dfrac{\sum d_i}{n}$ & $\bar{X} = A + h.\dfrac{\sum fu}{\sum f}$

(2). Median, $Q = L + \dfrac{i}{f}\left(\dfrac{n}{2} - c\right)$

(3). Mode, $Z = L_1 + \dfrac{f_m - f_1}{2f_m - f_1 - f_2}$

(4). Standard Deviation, $\sigma = \sqrt{\dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2}$

and $\sigma = i \times \sqrt{\dfrac{\sum fu^2}{\sum f} - \left(\dfrac{\sum fu}{\sum f}\right)^2}$

Calculus

(1). $y = x^n, \quad y' = nx^{n-1}$

(2). $y = cf(x), \quad y' = cf'(x)$

(3). $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

(4). $\dfrac{d}{dx}(u \pm v) = \dfrac{du}{dx} \pm \dfrac{dv}{dx}$

(5). $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

(6). $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

(7). $\int x^n dx = \dfrac{x^{n+1\rightarrow}}{n+1} + c, \; n \neq -1$

(8). $\int [f(x)]^n f'(x)dx = \dfrac{[f(x)]^{n+1}}{n+1}, n \neq -1$

Trigonometry

(1). $\sin^2\theta + \cos^2\theta = 1$

(2). $\sec^2\theta = 1 + \tan^2\theta$

(3). $\csc^2\theta = 1 + \cot^2\theta$

(4). $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

(5). $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

(6). $\tan(A \pm B) = \dfrac{\tan A \mp \tan B}{1 \mp \tan A\tan B}$

(7). $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(8). $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(9). $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

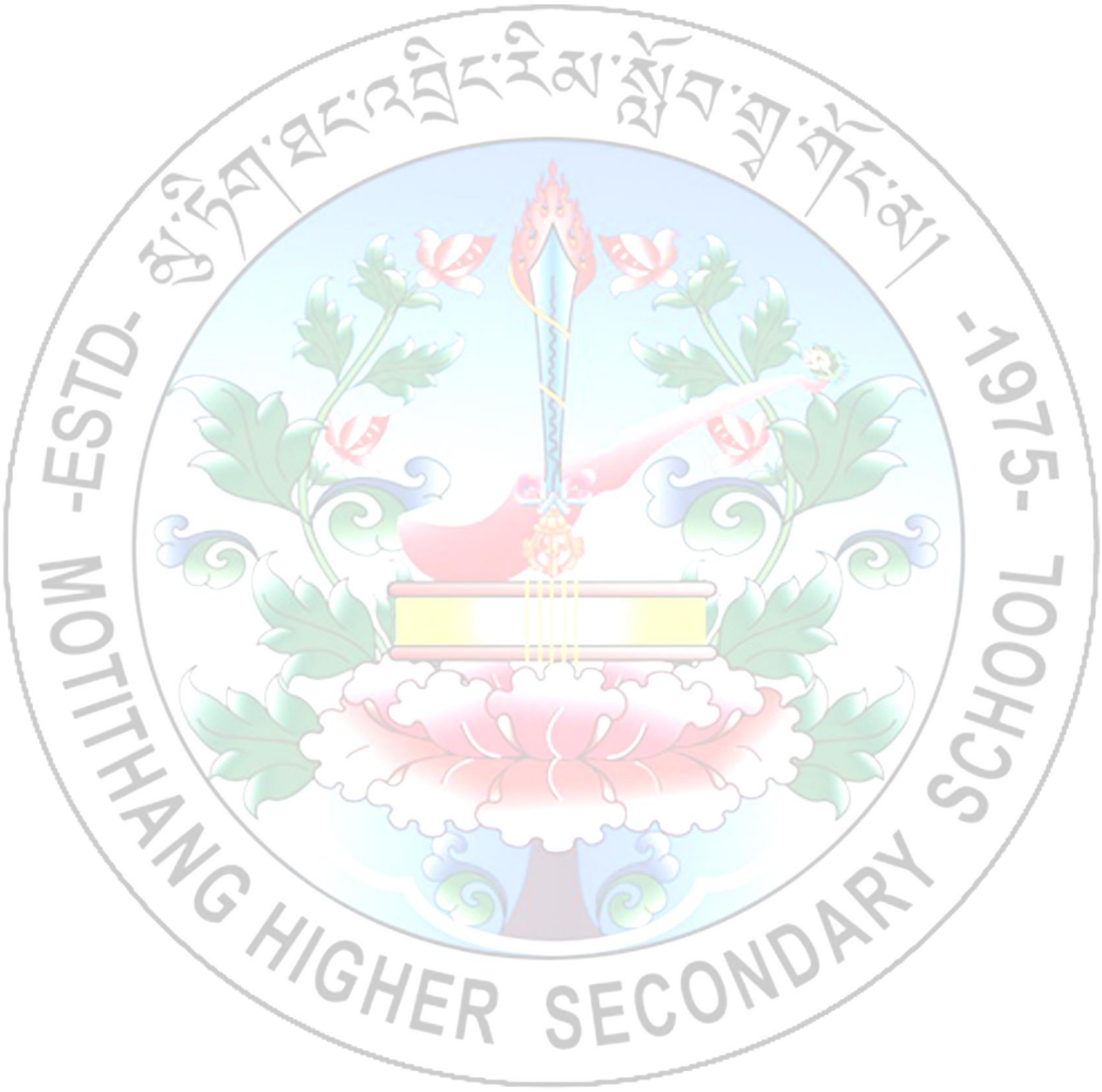
(10). $\cos C - \cos D = 2\sin\dfrac{C+D}{2}\sin\dfrac{D-C}{2}$

(11). $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(12). $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(13). $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(14). $2\sin A\sin B = \cos(A-B) - \cos(A+B)$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