



ཤེས་རིག་ལྷན་ཁག།
 ལྷ་ཉིག་ཐང་འབྲིང་རིམ་སློབ་གྲྭ་གོང་མ།



MOTITHANG HIGHER SECONDARY SCHOOL

THIMPHU THROMDE

“Every child is **inspired** to learn and **empowered** with **wisdom** to excel in life”

MID-TERM EXAMINATIONS, 2019

Mathematics

Reading Time: 15 mins

Class XII Sci

Writing Time: 3 hours

Date:

Full Marks: 100

Name:..... Roll No.Class:..... Sec:.....

Invigilator's initial

Questions	For Teacher's Use Only																											
	Section A Q1	Section B																										
		Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14														
Marks	30	3	4	4	3	5	2	3	4	4	3	5	2	2	5	4	3	4	3	4	3	3	4	3	4	4	3	
Award																												
Teacher's initial																												
Total Marks Awarded																												

Grand Total

READ THE FOLLOWING DIRECTIONS CAREFULLY:

1. Do not write for the first **15 minutes**. This time is to be spent reading the questions.
2. After having read the questions, you will be given **3 hours** to answer all questions.
3. This paper comprises of **2 Sections**. Answer **Question 1** from Section A and **10 questions** from Section B
4. All working, including rough work, should be done on the same sheet adjacent to the rest of the answer. The intended marks for questions or parts of questions are given in brackets [].
5. The use of calculator (fx-82/ fx-100) is allowed.
6. Remember to write quickly but neatly.

SECTION A

(Answer **All** questions)

Direction: For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there is more than one choice circled, NO score will be awarded.

Question 1

[2×15=30]

i) In how many ways can four boys and two girls be arranged so that no two girls are sitting together?

- A. $4! \times 60$ B. $4! \times 20$ C. $2! 4!$ D. $2! \times 12$

ii) The principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is

- A. $\frac{\pi}{3}$ B. $\frac{2\pi}{3}$ C. $\frac{\pi}{6}$ D. $\frac{5\pi}{6}$

iii) The derivative of $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \text{to } \infty}}$ is

- A. y B. \sqrt{x} C. x^2 D. $\frac{1}{2y-1}$

iv) $\int \frac{\tan^{-1} x}{1+x^2} dx$ is

- A. $\tan^{-1} x + c$ B. $\sec^2 x + c$ C. $\frac{(\tan^{-1} x)^2}{2} + c$ D. $\log x + c$

v) What is the nature of the solution for the given system?

$$4x - 2y = 6$$

$$2x - y = 3$$

- A. Unique solution B. Trivial solution C. Infinitely many solutions D. No solution

vi) If $f'(x) = 3x^2 - \frac{2}{x^3}$ and $f(1) = 0$, then $f(x)$ is

- A. $x^3 + \frac{1}{x^2}$ B. $x^3 + \frac{1}{x^2} - 2$ C. $x^3 + \frac{1}{x^2} + 2$ D. x^3

vii) The derivative of $(x^x)^x$ w. r. t. x is

- A. $x^{x^2+1}(1+2\log x)$ B. $x^{x^2}(1+2\log x)$ C. $x^{x^2}(2\log x)$ D. $x(1+2\log x)$

viii) How many four digit odd integers can be formed using 0, 1, 2, 3, 4, 5 ?

- A. 60 B. 540 C. 216 D. 1296

ix) The derivative of $(\sin^2 x)$ w. r. t. $(\log x)^2$ is

- A. $\frac{\sin 2x}{\log x}$ B. $\frac{x \sin 2x}{\log x}$ C. $\frac{x \sin 2x}{2 \log x}$ D. $\frac{\sin 2x}{2 \log x}$

x) The simplified form of $\cos\left(\frac{1}{2}\cos^{-1}x\right)$ is

- A. $\sqrt{x+1}$ B. $\sqrt{\frac{x+1}{2}}$ C. x D. $2x$

xi) If $x = a\cos^3 t$ and $y = a\sin^3 t$, what is the value of $\frac{dy}{dx}$?

- A. $-\tan t$ B. $\tan t$ C. $-\cot t$ D. $\sin t$

xii) The value of $\int_{-4}^2 |x+3| dx$ is

- A. 13 B. 12 C. 11 D. 10

xiii) If $P(n, 6) = 3P(n, 5)$, what is the value of n ?

- A. 6 B. 7 C. 8 D. 9

xiv) $\int \log x dx$ is given by

- A. $\frac{1}{x} + c$ B. $\log x + c$ C. $\log x + x + c$ D. $x \log x - x + c$

xv) If $ax^2 + bx + 4$ attains its maximum value of -1 at $x=1$, then the values of 'a' and 'b' are given by

- A. 5, -5 B. 10, 10 C. 10, -5 D. 5, -10

SECTION B (70 Marks)

Answer any 10 questions. All questions in this section have equal marks.

Question 2

1. i) Find how many arrangements can be made with letters of the word ASSIGNMENT. [3]

ii) In how many of the arrangements the vowels occur together?

2. If $x^m y^n = (x+y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$ [4]

Question 3

1. A semi-circle in the form of a rectangle surmounts a window. The perimeter of the window is 20 m, find the dimensions of the window when the area is maximum. [4]

2. Find the value of the definite integral $\int_0^{\frac{\pi}{2}} \frac{\sec x \tan x}{1 + \sec^2 x} dx$ [3]

Question 4

1. Differentiate w. r. t. x [2+3]

i) $\log\{\log(\log x)\}$ ii) $\frac{x \sin^{-1} x}{\sqrt{1-x^2}}$

2. Find the value of k if the following system of equations is consistent.

[2]

$$2x - y + 3 = 0$$

$$kx - y + 1 = 0$$

$$5x - y - 3 = 0$$

Question 5

1. Solve for x

[3]

$$\tan^{-1}(2+x) + \tan^{-1}(2-x) = \tan^{-1}\frac{2}{3}$$

2. Evaluate $\int \frac{1}{x^3 + x^4} dx$

[4]

Question 6

1. A committee of 5 is to be formed from a group of 12 students consisting of 8 boys and 4 girls. In how many ways can the committee be formed if it consists of [4]
- i) exactly 3 girls and 2 boys.

ii) atleast 3 boys.

2. Differentiate $\frac{a^x}{1+a^x}$ w. r. t. a^x

[3]

Question 7

1. Solve the following system of equations using determinants.

[5]

$$x - y + 3z = 6$$

$$x + 3y - 3z = -4$$

$$5x + 3y + 3z = 10$$

2. Find the coordinates of the point on the curve $y = 3x^2 - 4x$, whose gradient is 14. [2]

Question 8

1. Prove that $\cos^{-1} \frac{12}{13} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{56}{65}$ [2]

2. If $y = \sin(m \sin^{-1} x)$, show that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0$ [5]

Question 9

1. Evaluate

i) $\int e^x \cos e^x dx$

ii) $\int \sec^4 x \tan x dx$

[4]

2. Show that matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + 7I = 0$. Hence find A^{-1} . [3]

Question 10

1. If ${}^nC_r : {}^nC_{r+1} = 1:2$ and ${}^nC_{r+1} : {}^nC_{r+2} = 2:3$, find the value of n and r. [4]

2. Find $\frac{dy}{dx}$, if $\sin(x+y) = \log(x+y)$

[3]

Question 11

1. Prove that a cylindrical vessel of given volume requires a least surface area when its height is twice its radius. [4]

2. Verify that $(AB)^t = B^t A^t$ if

[3]

$$A = \begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 1 \\ 2 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & -2 & 1 \\ -1 & 2 & 3 \\ -1 & -2 & 2 \end{bmatrix}$$

Question 12

1. If $x = a(\theta + \sin \theta)$ and $y = a(1 - \cos \theta)$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$ [3]

2. Without expanding the determinant, prove that
$$\begin{vmatrix} y+z & x+y & x \\ z+x & y+z & y \\ x+y & z+x & z \end{vmatrix} = x^3 + y^3 + z^3 - 3xyz. \quad [4]$$

Question 13

1. Evaluate $\int x^3 \tan^{-1} x \, dx$

[3]

2. Prove that $4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{70} + \tan^{-1} \frac{1}{99} = \frac{\pi}{4}$ [4]

Question 14

1. Prove that $\int_0^{\pi} \theta \sin^2 \theta \cos^2 \theta d\theta = \frac{\pi^2}{16}$ [4]

2. Evaluate $\tan \frac{1}{2} \left(\cos^{-1} \frac{\sqrt{5}}{3} \right)$

[3]

.....: ***ALL THE BEST*** :.....